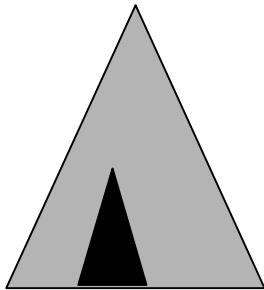


# Revocation and Tracing Schemes for Stateless Receivers

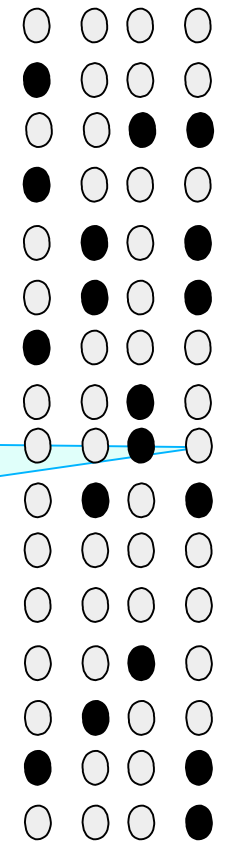
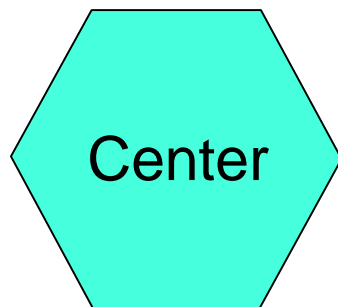
*Dalit Naor*  
*Moni Naor*  
*Jeff Lotspiech*



*IBM Almaden Research Lab*  
*Weizmann Institute, Stanford University*

# The Broadcast Encryption Problem

- **Center** transmits a message to a **large group**
- A **subset** of users is **revoked** and should not decrypt the message
  - subset changes **dynamically**
- Receivers are **Stateless**
  - independent of history
    - ▶ depend only on initial configuration
  - essential for "off-line" applications, useful otherwise



● revoked  
○ non-revoked

# Preliminaries

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- *Notation:*
  - $N$  - set of  $n$  users
  - $R$  - set of  $r$  users whose privileges are to be revoked;
- *Assumption: Stateless devices*
- *Goal: encrypt so that*
  - a non-revoked user can decrypt correctly
  - **No** coalition of revoked users (of an arbitrary size) can decrypt

# New Subset-Cover Revocation and Tracing Algorithms

---

*n* - total no. of users

*r* - no. revocations

*t* - no. of traitors (illegal users)

Scheme	Message Length	# Keys per Device	Processing Time	# decrypt.	Message Length for <i>t</i> traitors
<b>Complete Subtree</b>	$r \log n/r$	$\log n$	$\log \log n$	1	$t \log n$
<b>Subset Difference</b>	$2^{r-1}$ 1.25r (avg.)	$0.5 \log^2 n$	$\log n$ applications of a PRSG	1	5t

# Components

---

*A system consists of three parts:*

- 1. **Scheme Initiation** - a method to assign secret information to devices,  $I_u$  to user  $u \in N$ .*
- 2. **The broadcast algorithm** - given a message  $M$  and a set  $R$  of users to be revoked, output a ciphertext message to broadcast to all.*
- 3. **A decryption algorithm** -*
  - a non-revoked device should produce  $M$  from ciphertext.*
  - Decryption should be based on the current message and the secret information only (i.e. stateless).*
  - Impossible to produce  $M$  from ciphertext even when provided with the secret information of **all** revoked users.*

# Subset - Cover Framework

---

*An algorithm in the framework is defined by:*

- *Underlying collection of subsets (of devices)*  
 $S_1, S_2, \dots, S_W \quad S_j \subseteq N.$
- *Each subset  $S_j$  has a long-lived key  $L_j$  associated with it;*
  - *A device  $u \in S_j$  should be able to deduce  $L_j$  from its secret information  $I_u$ .*
- *Given a revoked set  $R$ , the non-revoked users  $N \setminus R$  are partitioned into **disjoint** subsets*  
 $S_{i_1}, S_{i_2}, \dots, S_{i_m} \quad (N \setminus R = \cup_j S_{i_j})$
- *a session key  $K$  is encrypted  $m$  times with  $L_{i_1}, L_{i_2}, \dots, L_{i_m}$*

# The Broadcast Algorithm

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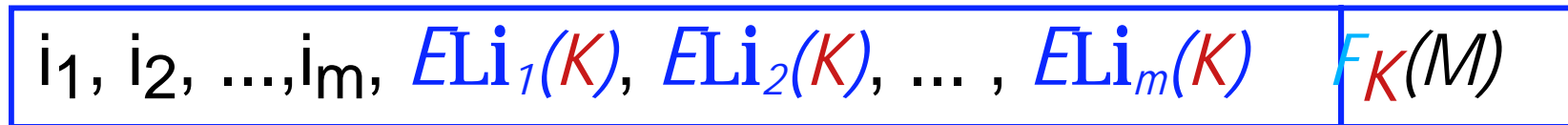
- Choose a session key  $K$
- Given  $R$ , find partition of  $N \setminus R$  into *disjoint* sets

$$S_{i_1}, S_{i_2}, \dots, S_{i_m}$$

$$N \setminus R = \cup_j S_j$$

with associated keys  $L_{i_1}, L_{i_2}, \dots, L_{i_m}$

- Encrypt message  $M$



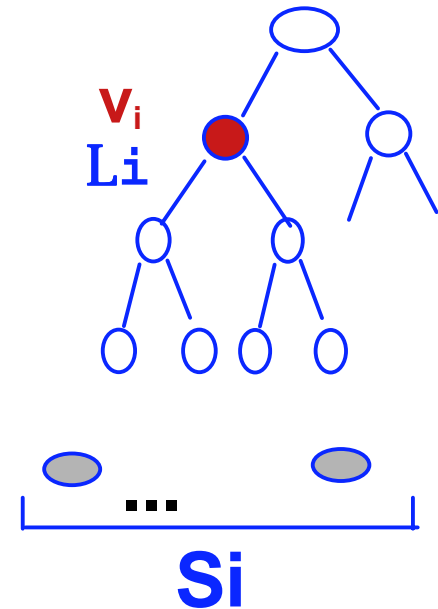
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Body

# The Complete Subtree Method

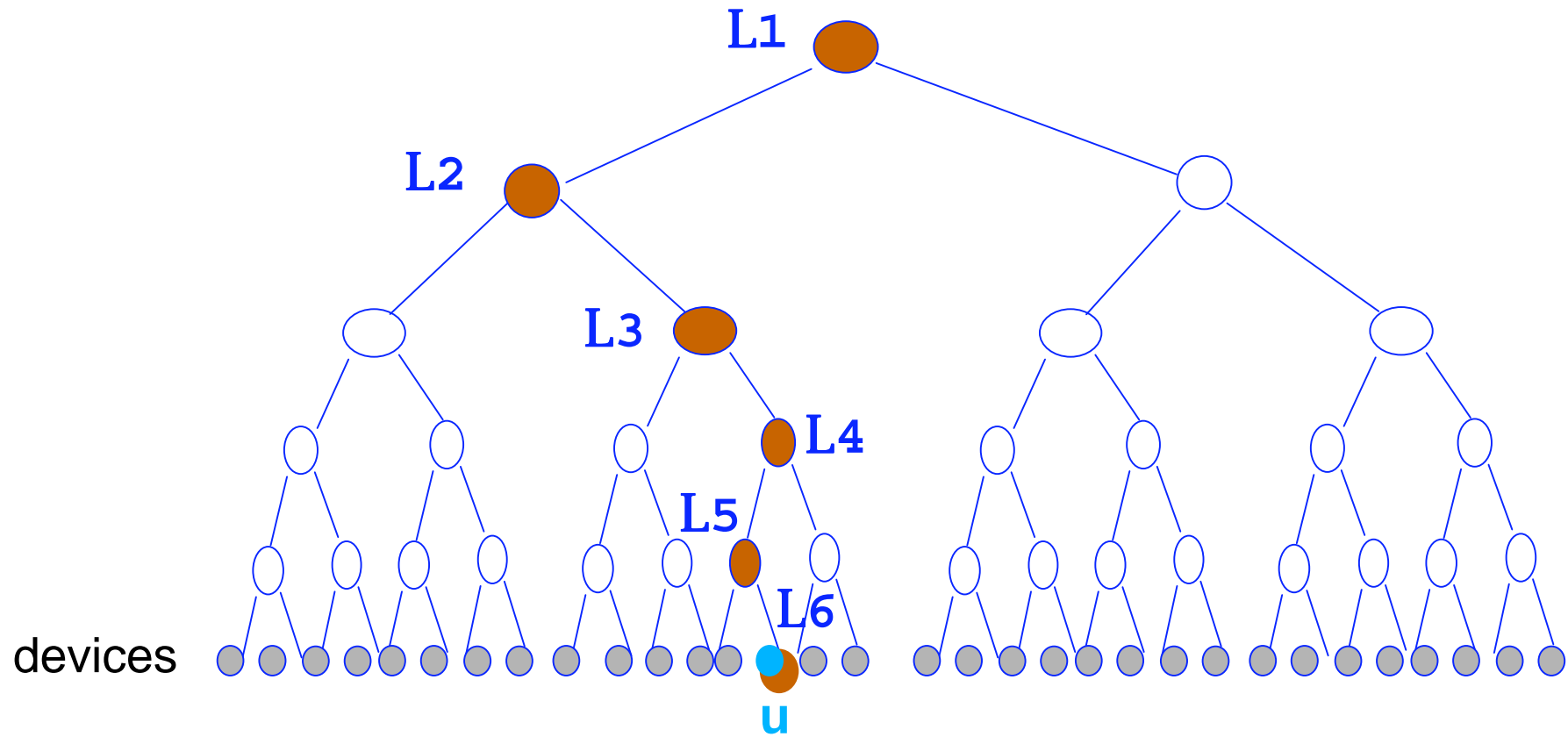
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- *Imagine a full-binary tree with  $N$  leaves*
  - ▶ *E.g. if  $n=2^{32}$ , a 32-levels complete binary tree*
- *Underlying Subsets  $S_1, S_2, \dots, S_W$* 
  - *for node  $v_i$  in the full tree,*
    - ▶  $S_i$  - *set of all leaves in the subtree of  $v_i$ .*
  - $w = 2n - 1$
- *Key assignment:*
  - *assign a key  $L_i$  to every node  $v_i$  in the tree*
- *Device keys:*
  - *store all  $\log n + 1$  keys along path to the root*
  - *E.g. if  $n=2^{32}$ , need 33 keys*



# Complete Subtree: Key Assignment

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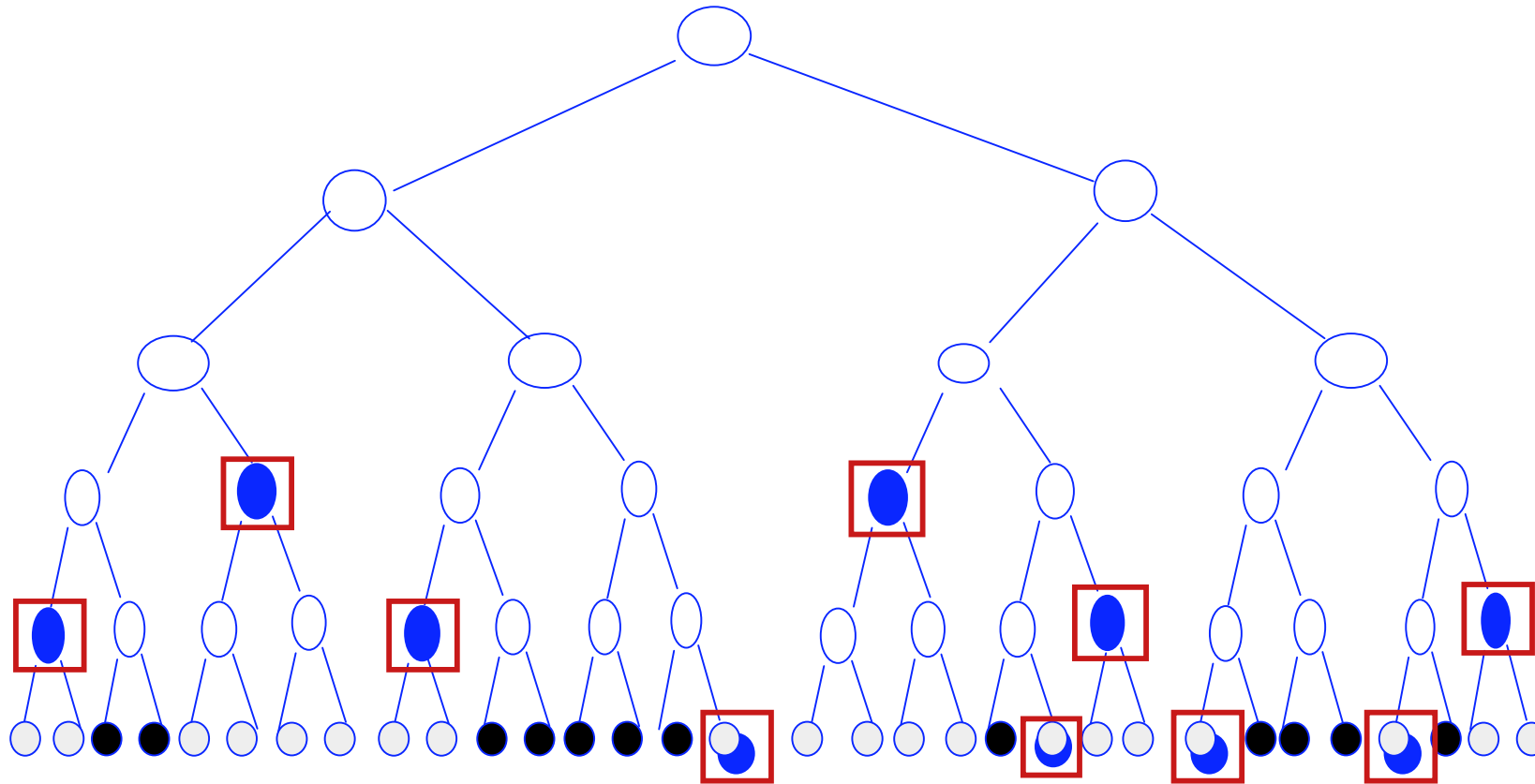


$$I_u = \{ L1, L2, L3, L4, L5, L6 \}$$

# Subset Cover of non-revoked devices

## Complete Subtree Method

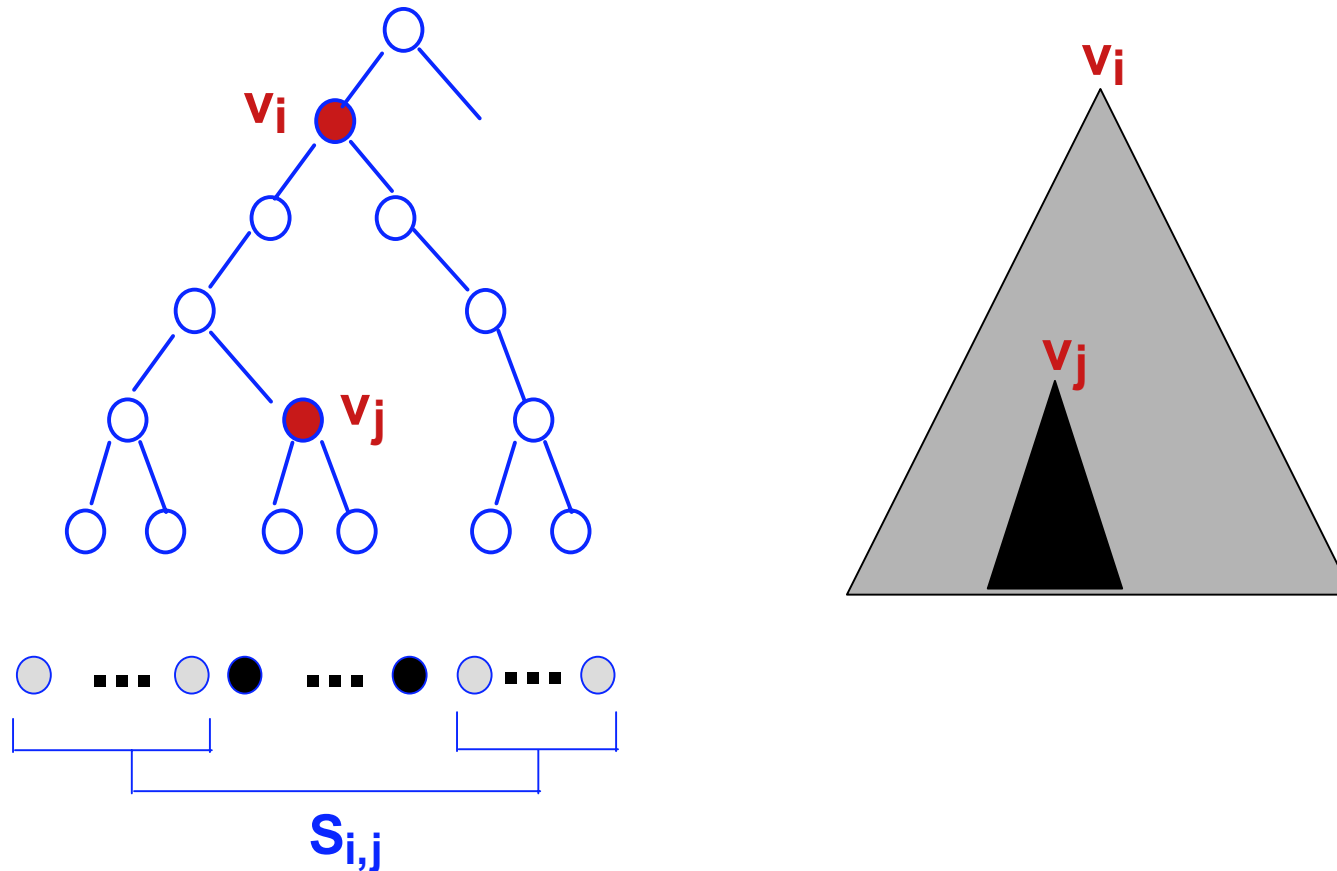
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- revoked
- non-revoked
- (blue circle) cover

# Subset Difference Definition

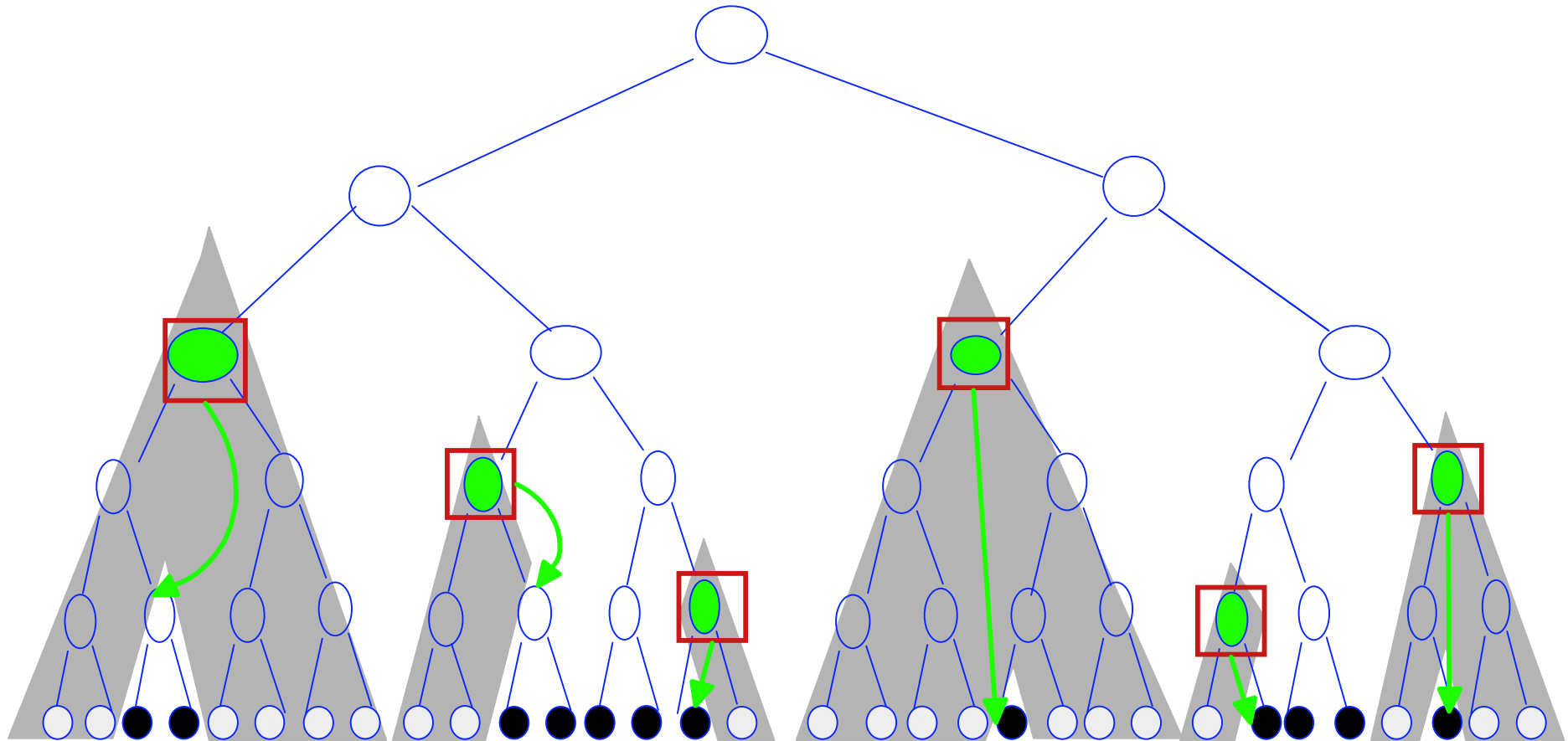
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$S_{i,j}$  = Set of all leaves in the subtree of  $v_i$  but not in  $v_j$

# Subset Cover of Non-Revoked Devices

## Subset-Difference Method



- revoked
- non-revoked
- cover

$$= S_{i,j} \quad \begin{matrix} V_i \text{ } \square \\ \curvearrowright \\ \text{ } \circ \text{ } V_j \end{matrix}$$

# Cover is Very Small !!

---

## Fundamental property:

*Size of the subset cover in the difference-subset method is*

*At most  $2r-1$  in the worst case*

*$1.25r$  in the average case !*

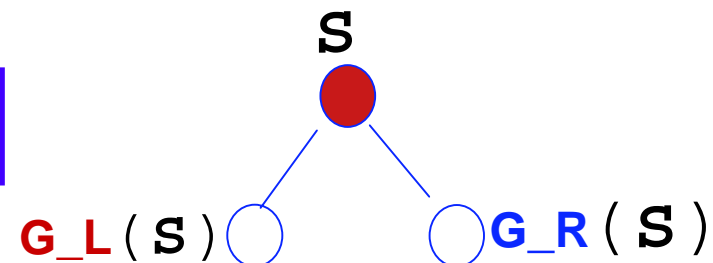
# Key-Assignment

## Subset-Difference Method

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- *Naive approach to the key assignment:*
  - ▶ assign a key  $L_{i,j}$  to every pair  $[v_i, v_j]$  in the tree
  - ▶ impractical: each device must store  $O(n)$  keys...
- Use  $G$ , a pseudo-random sequence generator that **triples** the input length ( $k \rightarrow 3k$ ) à la GGM
- Use  $G$  to derive a labeling process
  - $S$  - label @ node,
  - $G_L(S)$  - label @ left child,  $G_R(S)$  - label @ right child
  - $G_M(S)$  - key @ node.

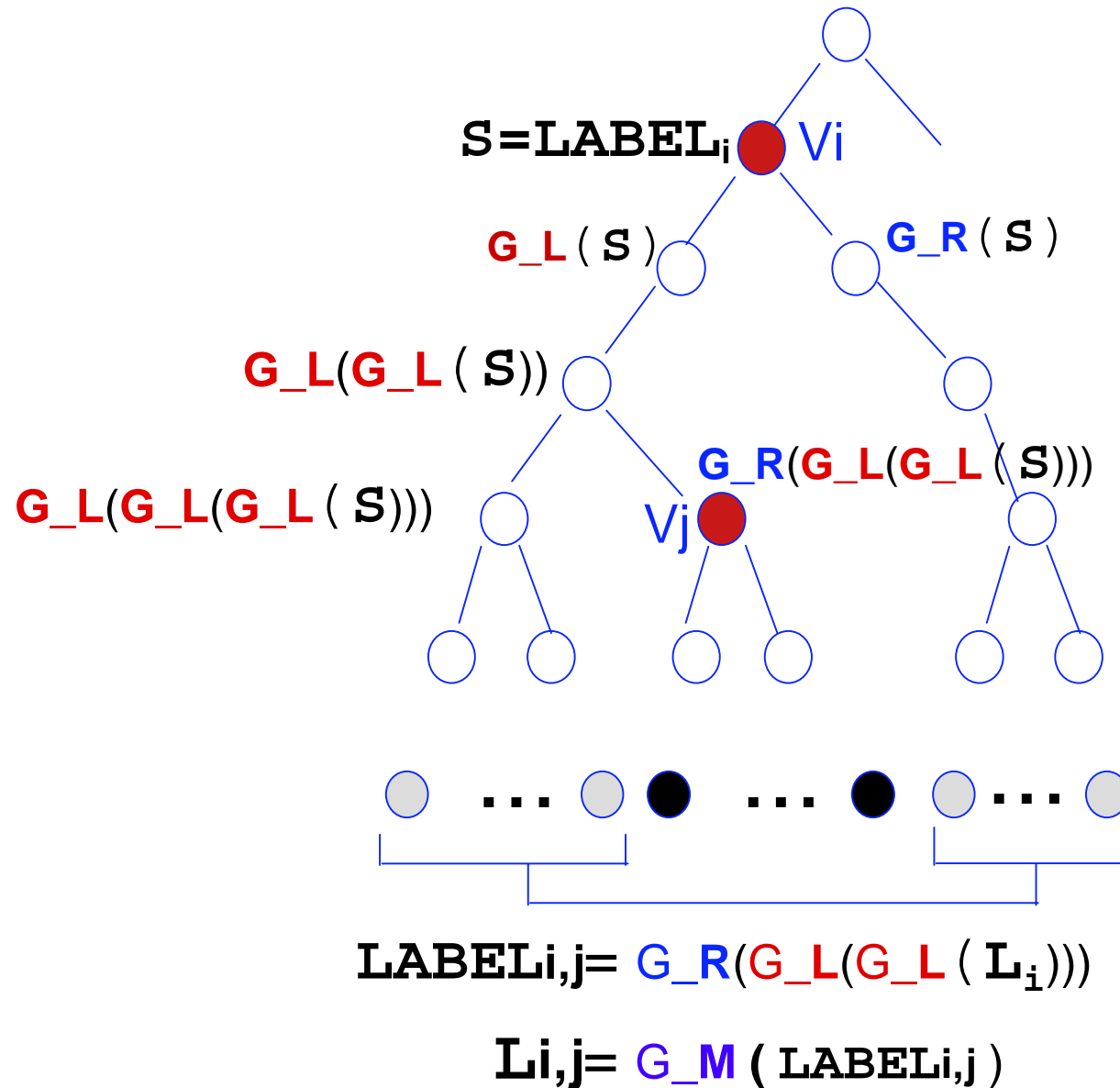
$$G(S) = \boxed{G_L(S) \quad G_M(S) \quad G_R(S)}$$



# Key-Assignment

## Subset-Difference Method

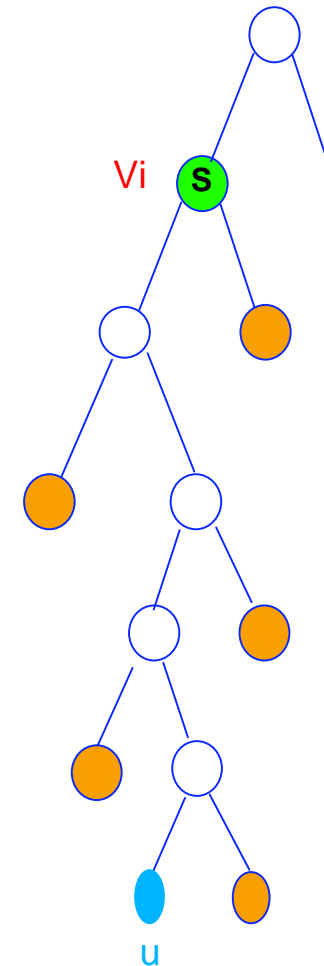
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# Providing Keys to Devices

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- A device corresponds to a leaf  $u$  in the tree
- For every  $V_i$  ancestor of  $u$  whose label is  $S$ 
  - $u$  receives all labels@nodes that are **hanging off the path** from  $V_i$  to  $u$ . These labels are all derived from  $S$ .
- $u$  can compute all keys of the sets it belongs to rooted at  $V_i$ , and only them.



# Only 13 bytes per Single Revocation

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- For  $N = 2^{32}$  and 7-bytes session-key
  - total of  $1.25 * 7 + 4 < 13$  bytes/revocations
  - 530 labels/device

